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13. SUPPLEMENTARY NOTES

14. ABSTRACT

This was an AFOSR Presidential Early Career Award for Scientists and Engineers (PECASE) in 1997. The research objectives were: (1) Chaotic scattering and applications to optical micro-lasing cavities; (2) Inducing chaos in electronic circuits; (3) Signal enhancement using stochastic resonance for anti-jamming; (4) Dynamics of semiconductor lasers and communicating with chaos. These were directly suggested by Air Force scientists or motivated by applications in Air Force missions. All objective were completed and results were published in over 70 refereed-journal papers. The PECASE helped support five post-doctoral fellows and graduate six Ph.D. and four Master students. During the project period the PI was elected as a Fellow of the American Physical Society, gave 50 invited lectures, and provided consultations to Air Force Research Laboratories.

15. SUBJECT TERMS

Nonlinear Dynamics, Chaos, Stochastic Resonance, Antijamming, Semiconductor Lasers

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Final Report

This report summarizes our activities under the Air Force Office of Scientific Research (AFOSR) Grant No. F49620-98-1-0400 entitled "Theoretical and Experimental Studies of Chaotic Dynamics with Defense Applications." This was an AFOSR Presidential Early Career Award for Scientists and Engineers (PECASE) in 1997. The duration of the award was from 4/1/1998 to 6/30/2003. The report is divided into the following Sections:

1. Objectives
2. Description of Achievements of Objectives
3. Accomplishments and New Findings (Selected)
4. Personnel Supported and Theses Supervised by PI
5. List of Publications
6. Interactions/Transitions
7. Honors

1 Objectives

1. Chaotic scattering and applications to optical microlasing cavities;
2. Inducing chaos in electronic circuits;
3. Signal enhancement using stochastic resonance for antijamming;
4. Dynamics of semiconductor lasers and communicating with chaos.

2 Description of Achievements of Objectives

All four Objectives have been achieved. Results were published in a number of refereed-journal papers.

2.1 Chaotic scattering and applications to optical microlasing cavities

The fundamental dynamics of chaotic scattering in two- and three-degrees of Hamiltonian systems, which include topological bifurcations and the effect of weak dissipation, were investigated. Guided by the theory of chaotic scattering, ray tracing in optical microlasing cavities was carried out, yielding an understanding of how geometrical deformations affect the Q-value and directionality of the lasing cavity. The effect of quantum-mechanical tunneling associated with nonhyperbolic chaotic scattering in semiconductor nanostructure (quantum dots) was disscovered and explained using a semiclassical theory.

Relevant publications are

- Y.-C. Lai, "Abrupt bifurcation to chaotic scattering with discontinuous change in fractal dimension," *Physical Review E (Rapid Communications)* **60**, R6283-R6286 (1999).
- Y.-C. Lai, K. Zyczkowski, and C. Grebogi, "Universal behavior in the parameter evolution of chaotic saddles," *Physical Review E* **59**, 5261-5265 (1999).
- K. Zyczkowski and Y.-C. Lai, "Devil-staircase behavior of dynamical invariants in chaotic scattering," *Physica D* **142**, 197-216 (2000).
- Y.-C. Lai, A. P. S. de Moura, and C. Grebogi, "Topology of high-dimensional chaotic scattering," *Physical Review E* **62**, 6421-6428 (2000).
- A. E. Motter and Y.-C. Lai, "Dissipative chaotic scattering," *Physical Review E (Rapid Communications)* **65**, 015205(1-4) (2002).
- A. Motter and Y.-C. Lai, "Dimension scaling of topological bifurcations in chaotic scattering," *Physical Review E (Rapid Communications)* **65**, 065201(1-4) (2002).
- Z. Liu and Y.-C. Lai, "Chaotic scattering in deformed optical microlasing cavities," *Physical Review E* **65**, 046204(1-5) (2002).
- A. P. S. de Moura, Y.-C. Lai, R. Akis, J. Bird, and D. K. Ferry, "Tunneling and nonhyperbolicity in quantum dots," *Physical Review Letters* **88**, 236804(1-4) (2002).

2.2 Inducing chaos in electronic circuits

This topic was suggested by Drs. Mike Harrison and Dave Dietz at AFRL, Kirtland AFB. During the project period, experimental electronic circuits including the Chua's circuit, the Rössler circuit, and the coupled Rössler circuits were set up, tested, and utilized to investigate problems on fundamental properties of chaos and induction of chaos in these circuits. Specific achievements are: (1) experimental observation of chaotic lag synchronization, (2) experimental observation and characterization of superpersistent chaotic transients and scaling law, (3) theoretical and numerical schemes of inducing chaos by using resonant perturbations, (4) scaling theory of noise-induced chaos and experimental verification, and (5) noisy scaling of statistical averages in chaotic systems and experimental verification.

These achievements were summarized in the following papers and report:

- S. Taherion and Y.-C. Lai, "Observability of lag synchronization in coupled chaotic oscillators," *Physical Review E (Rapid Communications)* **59**, R6247-R6250 (1999).
- S. Taherion and Y.-C. Lai, "Experimental observation of lag synchronization in coupled chaotic systems," *International Journal of Bifurcation and Chaos* **10**, 2587-2594 (2000).
- L. Zhu, A. Raghu, and Y.-C. Lai, "Experimental observation of superpersistent chaotic transients," *Physical Review Letters* **86**, 4017-4020 (2001).
- L. Zhu and Y.-C. Lai, "Experimental observation of generalized time-lagged chaotic synchronization," *Physical Review E (Rapid Communications)* **64**, 045205(1-4) (2001).
- Y.-C. Lai, Z. Liu, and A. P. S. de Moura, "Inducing chaos in nonlinear oscillators by resonant perturbations," Technical Report submitted to AFOSR (Dr. Arje Nachman) and AFRL (Dr. Mike Harrison), February 2002.
- Z. Liu, Y.-C. Lai, L. Billings, and I. B. Schwartz, "Transition to chaos in continuous-time random dynamical systems," *Physical Review Letters* **88**, 124101(1-4) (2002).
- Y.-C. Lai, Z. Liu, G. Wei, and C.-H. Lai, "Shadowability of statistical averages in chaotic systems," *Physical Review Letters* **89**, 184101(1-4) (2002).
- B. Xu, Y.-C. Lai, L. Zhu, and Y. Do, "Experimental characterization of transition to chaos in the presence of noise," *Physical Review Letters* **90**, 164101 (1-4) (2003).
- Y.-C. Lai, Z. Liu, L. Billings, and I. B. Schwartz, "Noise-induced unstable dimension variability and transition to chaos in random dynamical systems," *Physical Review E* **67**, 026210(1-17) (2003).
- L. Zhu, Y.-C. Lai, F. Hoppensteadt, and E. M. Bollt, "Numerical and experimental investigation of the effect of filtering on chaotic symbolic dynamics," *Chaos* **13**, 410-419 (2003).

2.3 Signal enhancement using stochastic resonance for antijamming

The Objective was to test and idea of using stochastic resonance to suppress jamming. The idea was conceived by Dr. Arje Nachman at AFOSR. During the project period, a scheme based on excitable dynamical systems was proposed and numerically tested for enhancing both periodic and aperiodic signals under jamming by using stochastic resonance. The aperiodic signals tested include AM, FM, and chaotic signals. As a related problem, theoretical, numerical, and experimental studies of coherence resonance (enhancement of temporal regularity by noise) were carried out. Enhancement of chemical reaction in chaotic flows by noise was discovered and explained.

Publications are

- Z. Liu and Y.-C. Lai, "Coherence resonance in coupled chaotic oscillators," *Physical Review Letters* **86**, 4737-4740 (2001).

- Y.-C. Lai and Z. Liu, "Noise-enhanced temporal regularity in coupled chaotic oscillators," *Physical Review E* **64**, 066202(1-9) (2001).

- Z. Liu, Y.-C. Lai, and A. Nachman, "Enhancement of noisy signals by stochastic resonance," *Physics Letters A* **297**, 75-80 (2002).

- L. Zhu, Y.-C. Lai, Z. Liu, and A. Raghu, "Can noise make non-bursting chaotic systems more regular?" *Physical Review E (Rapid Communications)* **66**, 015204(1-4) (2002).

- Z. Liu, Y.-C. Lai, and J. M. Lopez, "Noise-induced enhancement of chemical reactions in chaotic flows," *Chaos* **12**, 417-425 (2002). [This work was featured in AIP (American Institute of Physics) News Updates in July 2002. It was also named an outstanding paper for the journal CHAOS in the AIP's 2002 Annual Report.]

- Z. Liu, Y.-C. Lai, and A. Nachman, "Enhancement of detectability of noisy signals by stochastic resonance in arrays," *International Journal of Bifurcation and Chaos*, accepted.

- Y.-C. Lai, Z. Liu, and A. Nachman, "Aperiodic stochastic resonance and phase synchronization," submitted to *Physics Letters A*.

- Y.-C. Lai, Z. Liu, A. Nachman, and L. Zhu, "Suppression of jamming in excitable systems by aperiodic stochastic resonance," submitted to *International Journal of Bifurcation and Chaos.*

2.4 Dynamics of semiconductor lasers and communicating with chaos

Research on semiconductor lasers was done in close collaboration with Air Force scientists (Drs. A. Gavrielides and V. Kovanis) at the Nonlinear Optics Center, AFRL, Kirtland AFB. Problems investigated included the dynamical origin of low-frequency fluctuations in semiconductor lasers, fundamental regular dynamical invariant sets associated with these fluctuations, analysis of a reduced model for understanding the fluctuations, complicated basin structure in semiconductor lasers, and a possible scheme to suppress the low-frequency fluctuations. In communicating with chaos, the issues of high-dimensional symbolic dynamics, transient chaos, determination of generating partition using unstable periodic orbits, and data analysis using symbolic dynamics were explored.

The results are summarized in the following publications:

- Y.-C. Lai, E. Bollt, and C. Grebogi, "Communicating with chaos using two-dimensional symbolic dynamics," *Physics Letters A* **255**, 75-81 (1999).

- Y.-C. Lai "Encoding digital information using transient chaos," *International Journal of Bifurcation and Chaos* **10**, 787-795 (2000).

- R. L. Davidchack, Y.-C. Lai, E. Bollt, and M. Dhamala, "Estimating the generating partition of chaotic systems by unstable periodic orbits," *Physical Review E* **61**, 1353-1356 (2000).

- R. L. Davidchack, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Dynamical origin of low-frequency fluctuations in external cavity semiconductor lasers," *Physics Letters A* **267**, 350-356 (2000).

- R. L. Davidchack, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Chaotic transitions and low-frequency fluctuations in semiconductor lasers with optical feedback," *Physica D* **145**, 130-143 (2000).

- E. Bollt, T. Stanford, Y.-C. Lai, and K. Zyczkowski, "Validity of threshold-crossing analysis of symbolic dynamics from chaotic time series," *Physical Review Letters* **85**, 3524-3527 (2000).
- E. Bollt, T. Stanford, Y.-C. Lai, and K. Zyczkowski, "What symbolic dynamics do we get with a misplaced partition? - On the validity of threshold crossings analysis of chaotic time series," *Physica D* **154**, 259-286 (2001).
- R. L. Davidchack, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Regular dynamics of low-frequency fluctuations in semiconductor lasers with optical feedback," *Physical Review E* **63**, 056206(1-6) (2001).
- A. Prasad, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Low-frequency fluctuations in external-cavity semiconductor lasers: understanding based on a reduced Lang-Kobayashi model," *Journal of Optics B: Quantum and Semiclassical Optics* **3**, 242-250 (2001).
- A. Prasad, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Complicated basins in coupled external-cavity semiconductor lasers," *Physics Letters A* **314**, 44-50 (2003).
- A. Prasad, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Amplitude modulation in coupled external-cavity semiconductor lasers," *Physics Letters A*, in press.

3 Accomplishments and New Findings (Selected)

3.1 Objective 1: Chaotic scattering and applications to optical microlasing cavities

3.1.1 Crisis in chaotic scattering

A crisis in chaotic scattering is characterized by the merging of two or more nonattracting chaotic saddles. The fractal dimension of the resulting chaotic saddle increases through the crisis. We formulated a rigorous analysis for the behavior of dynamical invariants associated with chaotic scattering by utilizing a representative model system that captures the essential dynamical features of crises. Our analysis indicated that the fractal dimension and other dynamical invariants are a devil-staircase type of functions of the system parameter. The results can also serve as a rigorous base for similar devil-staircase behaviors observed in the parametric evolution of chaotic saddles of general dissipative dynamical systems and those arising in communicating with chaos.

- Y.-C. Lai, K. Zyczkowski, and C. Grebogi, "Universal behavior in the parameter evolution of chaotic saddles," *Physical Review E* **59**, 5261-5265 (1999).
- K. Zyczkowski and Y.-C. Lai, "Devil-staircase behavior of dynamical invariants in chaotic scattering," *Physica D* **142**, 197-216 (2000).

3.1.2 Topology of high-dimensional chaotic scattering

A fundamental problem in the study of chaotic scattering is to understand how the scattering characteristics change as a system parameter of physical interest changes. In this regard, most previous research had been on two-degree-of-freedom Hamiltonian systems or on three-degree-of-freedom Hamiltonian systems but with hard-wall potentials. We investigated chaotic scattering in three-degree-of-freedom Hamiltonian systems consisting of physically realistic soft potentials. Another motivation came from the desire to understand the scattering dynamics of particles by molecules in three-dimensional physical space and the ray dynamics in optical microlasing cavities. Our findings were: (1) the topology of the chaotic scattering dynamics can undergo a sudden change, called metamorphoses, as a system parameter (e.g., energy) changes continuously, (2) at a topological metamorphosis, the behavior of the fractal dimension of the chaotic saddle can change characteristically, and (3) chaotic scattering can occur in energy regimes where it is not possible in the corresponding planar scattering system.

- Y.-C. Lai, A. P. S. de Moura, and C. Grebogi, "Topology of high-dimensional chaotic scattering," *Physical Review E* **62**, 6421-6428 (2000).

3.1.3 Abrupt bifurcation to chaotic scattering

One of the major routes to chaotic scattering is abrupt bifurcation by which a nonattracting chaotic saddle is created, as a system parameter changes through a critical value. In a previously investigated case, however, the fractal dimension of the set of singularities in the scattering function changes continuously through the bifurcation. In this work, a new type of abrupt bifurcation to chaotic scattering was identified and analyzed, by which the physically relevant dimension changes discontinuously at the bifurcation. The bifurcation was illustrated by using a class of open Hamiltonian systems consisting of Morse potential hills.

- Y.-C. Lai, "Abrupt bifurcation to chaotic scattering with discontinuous change in fractal dimension," *Physical Review E (Rapid Communications)* **60**, R6283-R6286 (1999).

3.1.4 Chaotic scattering in optical microlasing cavities

Optical processes in microcavities are a subject of recent interest due to their relevance to designing novel microlasers, to nonlinear optics, and to atomic physics. One appealing feature of these dielectric cavities is that they can support high-Q whispering-gallery (WG) modes of operation which is due, clasically, to total internal reflection of the trapped light. However, microlasers require a high degree of directionarity of the emitted light to operate, which can be achieved by deforming the shape of the cavity from the ideal spherical or cylindrical geometry. Such deformations of cavities can lead to the breakdown of total internal reflections and, consequently, to Q-spoiling of the WG modes. The traditional wave perturbation theory cannot be employed to treat the problem of Q-spoiling because the deformations can be quite large. Existing works had demonstrated that many features of the Q-spoiling phenomenon can actually be understood by taking the classical approach of ray tracing, the dynamics of which is typically chaotic.

We studied the common class of dielectric optical microlasing cavities with quadrupolar deformations and addressed the question of the maximally allowed amount of deformation for both high-Q operation and a high degree of directionality of light emission. Our approach was to compute the probability for light rays to be trapped in the cavity by examining chaotic scattering dynamics in the classical phase space. We developed a dynamical criterion for high-Q operation and introduced a measure to quantify the directionality of the light emission. Our results suggested that high-Q and directionality can be achieved simultaneously in a wide range of the deformation parameter.

- Z. Liu and Y.-C. Lai, "Chaotic scattering in deformed optical microlasing cavities," *Physical Review E* **65**, 046204(1-5) (2002).

3.1.5 Effect of weak dissipation on chaotic scattering

In chaotic scattering, most existing works had been on Hamiltonian or conservative systems. In a realistic situation, a small amount of dissipation can be expected. We showed that weak dissipation can have a metamorphic consequence on nonhyperbolic chaotic scattering in the sense that the physically important particle-decay law is altered, no matter how small the amount of dissipation is. The previous result about the unity of the fractal dimension of the set of singularities in scattering functions, a major claim about nonhyperbolic chaotic scattering, may then not be observable.

- A. E. Motter and Y.-C. Lai, "Dissipative chaotic scattering," *Physical Review E (Rapid Communications)* **65**, 015205(1-4) (2002).

3.1.6 Tunneling and nonhyperbolicity in quantum dots

Electronic transport in semiconductor nanostructures is a frontier problem in condensed matter physics and nonlinear science. In sub-micron scales, quantum interference plays a fundamental role, giving rise to such phenomena as conductance fluctuations and the Aharonov-Bohm effect. A particularly important class of nanostructures is represented by the 2-Dimensional Electron Gas (2DEG) quantum dots. In these systems, the carriers (typically electrons) are restricted to move on a plane that lies near the interface between two different semiconductors. Applying voltage to contact gates deposited above the junction allows for the construction of sub-micron-sized 2D cavities in which electrons are scattered. Furthermore, in 2DEGs both the mean free path and the coherence lengths are typically much larger than the cavity length at milli-Kelvin temperatures. For low currents, the transport characteristics of the quantum dot (such as the conductance) are determined by the approximately ballistic and coherent motion of single electrons in the cavity. Semiclassical theory can thus be applied, and one can expect that the classical electron dynamics, e.g., whether the scattering is regular or chaotic, will play a major role in the transport. A popular approach had been to assume that the underlying classical scattering dynamics is completely chaotic (or hyperbolic), and then used the Random Matrix Theory (RMT), which predicts universal conductance fluctuations with a Lorentzian correlation function, as a parameter of the system is varied, such as an external magnetic field or the gate voltage. A fundamental difficulty with the RMT approach is that typical systems have a nonhyperbolic dynamics, with regions of chaotic scattering coexisting with non-escaping Kolmogorov-Arnold-Moser (KAM) islands surrounding stable orbits in the phase space.

Our belief was that many major features in electronic transport in realistic quantum dots are not explainable by the usual semiclassical approach, due to the contributions of the quantum-mechanical tunneling of the electrons through the KAM islands. We showed that dynamical tunneling gives rise to a set of resonances characterized by two quantum numbers, which leads to conductance oscillations and concentration of wave functions near stable and unstable periodic orbits. Experimental results agree very well with our theoretical predictions, indicating that tunneling has to be taken into account to understand the physics of transport in generic nanostructures.

- A. P. S. de Moura, Y.-C. Lai, R. Akis, J. Bird, and D. K. Ferry, "Tunneling and nonhyperbolicity in quantum dots," *Physical Review Letters* **88**, 236804(1-4) (2002).

3.2 Objective 2: Inducing chaos in electronic circuits

3.2.1 Inducing chaos in nonlinear oscillators by resonant perturbations

In 1998, Drs. Mike Harrison and Dave Dietz at the Air Force Research Laboratory (AFRL) in Kirtland Air Force Base suggested the following problem: Given a nonlinear oscillator such as an electronic circuit that operates in a regular and stable state, would it be possible to deliver small external perturbations to drive the oscillator into a chaotic state? The problem was motivated by Air Force's interest to defeat electronic tracking and guidance systems such as those found in surface-to-air missiles. If chaotic dynamics can be induced in some important parts of the tracking and guidance systems by using external excitations such as microwave oscillations, it is possible that the missile carrying these systems will fail to reach its target. This is so because the normal operational state of the electronic circuitry in such a system is typically stable, while in a chaotic state, the system will become "confused" and therefore fail in its intended mission. A potential advantage to use chaos is that the absorbed energy necessary to induce chaotic behavior can be much less than that required to simply "overpower" the same electronics.

We developed a theoretical strategy, based on the principle of resonant perturbations, to achieve the goal of inducing chaos. The general assumption is that either such an oscillator is incapable of generating any chaotic behavior, or it is far away from any chaotic regime in the parameter space. Our idea was to deliver judiciously chosen, small perturbations to drive the system into higher and higher resonant states in relatively short time. The perturbations can be a sinusoidal microwave field with time-varying frequency and phase, and how they vary is determined by a real-time measured signal emitted from the oscillator. That is, the perturbations can be computed based on a measured time series from the oscillator. We demonstrated numerically that the idea can indeed work, and we are hopeful that it can be implemented in laboratory experiments and eventually be applied to real-world situations.

- Y.-C. Lai, Z. Liu, and A. P. S. de Moura, "Inducing chaos in nonlinear oscillators by resonant perturbations," Technical Report submitted to AFOSR (Dr. Arje Nachman) and AFRL (Dr. Mike Harrison), February 2002.

3.2.2 Chaotic lag synchronization in electronic circuits

Lag synchronization means that the dynamical variables of two coupled, nonidentical chaotic oscillators can be synchronized but with a time delay relative to each other. We investigated experimentally, numerically, and theoretically to what extent lag synchronization can be observed in physical systems where noise is inevitable. Our measurements and analyses suggested that lag synchronization is typically destroyed when noise is comparable to the amount of average system mismatch. At small noise levels, lag synchronization occurs in an intermittent fashion. We provided a detailed experimental analysis and a theoretical explanation for the observed intermittent behavior.

- S. Taherion and Y.-C. Lai, "Observability of lag synchronization in coupled chaotic oscillators," *Physical Review E (Rapid Communications)* **59**, R6247-R6250 (1999).
- S. Taherion and Y.-C. Lai, "Experimental observation of lag synchronization in coupled chaotic systems," *International Journal of Bifurcation and Chaos* **10**, 2587-2594 (2000).
- L. Zhu and Y.-C. Lai, "Experimental observation of generalized time-lagged chaotic synchronization," *Physical Review E (Rapid Communications)* **64**, 045205(1-4) (2001).

3.2.3 Experimental observation of superpersistent chaotic transients

Transient chaos is ubiquitous in nonlinear dynamical systems. In such a case, dynamical variables of the system behave chaotically for a finite amount of time before settling into a final state that is usually not chaotic. A common situation for transient chaos to arise is where the system undergoes a crisis at which a chaotic attractor collides with the basin boundary separating it and another coexisting attractor. After the crisis, the chaotic attractor is destroyed and converted into a nonattracting chaotic saddle. Dynamically, a trajectory then wanders in the vicinity of the chaotic saddle for a period of time before approaching asymptotically the other attractor. Chaotic transients of this sort are not super persistent in the sense that their average lifetimes scale with the system parameter only algebraically.

There exists, however, a distinct class of chaotic transients that are superpersistent in the sense that their transient lifetimes scale with a parameter in an exponentially manner, with the exponent approaching asymptotically infinity as the parameter approaches a critical value. Physically, this means that the transient lifetime is significantly longer than those associated with "regular" chaotic transients characterized by an algebraic scaling law. Because of this superpersistent nature of transient chaos, the asymptotic attractor of the system is practically unobservable.

While regular chaotic transients had been observed in experiments, there had been no direct experimental verification of superpersistent chaotic transients. Because of the extremely long nature of these transients, it is highly nontrivial to observe and quantify them in laboratory experiments. We made the first experimental observation of superpersistent chaotic transients by investigating the effect of noise on phase synchronization in coupled chaotic electronic circuits and obtained the scaling relation that is characteristic of those extremely long chaotic transients.

- L. Zhu, A. Raghu, and Y.-C. Lai, "Experimental observation of superpersistent chaotic transients," *Physical Review Letters* **86**, 4017-4020 (2001).

3.2.4 Experimental investigation of effect of filtering on chaotic symbolic dynamics

Experimentally measured signals are either naturally or intentionally filtered, the former can be attributed to the limitation of the measuring instruments while the latter is due to the necessity to remove undesirable frequency components for signal processing. Another area to which filtering is relevant is transmission of chaotic signals through a physical medium. For instance, in a communication application, a chaotic waveform is transmitted through a channel. Because of the finite bandwidth of the channel, the transmission is equivalent to a filtering process. Most existing works on the effect of filtering on chaotic signals had dealt with how filtering changes the fractal dimensions, with well-known results such as the dimension increase caused by filtering. The focus of our investigation was on the symbolic-dynamics aspect of chaotic signals. Suppose a dynamical system generates a chaotic signal with a well-defined symbolic dynamics, and suppose this signal is filtered. We asked, to what extent is the chaotic symbolic dynamics affected by filtering? We focused on the topological entropy and the bit-error rate, two quantities characterizing the symbolic dynamics. Theoretical considerations indicated that the topological entropy is invariant under linear filtering, which we verified using numerical computations and experiments with a chaotic electronic circuit. Our results suggested that in practical situations, with reasonable care, the estimated topological entropy can be preserved and the bit-error rate can be maintained at low values for a wide range of the filtering parameter.

- L. Zhu, Y.-C. Lai, F. Hoppensteadt, and E. M. Bollt, "Numerical and experimental investigation of the effect of filtering on chaotic symbolic dynamics," *Chaos* **13**, 410-419 (2003).

3.2.5 Noise-induced chaos and scaling laws

We recently obtained results concerning the transition to chaos in random dynamical systems. In particular, situations were considered where a periodic attractor coexists with a nonattracting chaotic saddle, which can be expected in any periodic window of a nonlinear dynamical system. Under noise, the asymptotic attractor of the system can emulate chaotic behavior, as characterized by the appearance of a positive Lyapunov exponent. Generic features of the transition include: (1) the noisy chaotic attractor is necessarily nonhyperbolic as there are periodic orbits embedded in it with distinct numbers of unstable directions (unstable dimension variability), and this nonhyperbolicity develops as soon as the attractor becomes chaotic; (2) for systems described by differential equations, the unstable dimension variability destroys the neutral direction of the flow in the sense that there is no longer a zero Lyapunov exponent after the noisy attractor becomes chaotic; (3) the largest Lyapunov exponent becomes positive from zero in a continuous manner, and its scaling with the variation of the noise amplitude is algebraic. Formulas for the scaling exponent were derived in all dimensions. Numerical support using both low and high-dimensional systems and experimental verification using chaotic electronic circuits were obtained.

- Z. Liu, Y.-C. Lai, L. Billings, and I. B. Schwartz, "Transition to chaos in continuous-time random dynamical systems," *Physical Review Letters* **88**, 124101(1-4) (2002).
- B. Xu, Y.-C. Lai, L. Zhu, and Y. Do, "Experimental characterization of transition to chaos in the presence of noise," *Physical Review Letters* **90**, 164101 (1-4) (2003).
- Y.-C. Lai, Z. Liu, L. Billings, and I. B. Schwartz, "Noise-induced unstable dimension variability and transition to chaos in random dynamical systems," *Physical Review E* **67**, 026210(1-17) (2003).

3.2.6 Scaling law of statistical averages with noise in chaotic systems

A frequent task in computational and experimental sciences is to compute or measure statistical averages of some physical observables. A fundamental question is whether these averages can be computed or measured reliably. This is particularly relevant when the system under investigation exhibits chaos, for which numerical trajectories are not always shadowable by true trajectories. Shadowability of statistical averages has begun to be addressed. We recently identified a common situation in chaotic dynamics where statistical averages change with noise. In particular, we found that for chaotic systems in periodic windows, statistical averages typically scale algebraically with the noise amplitude. We derived formulas for the scaling exponent in all dimensions, provided extensive numerical support, and also presented experimental evidence to support the noisy scaling law using a chaotic electronic circuit. As periodic windows are common in nonlinear systems, the implication can be quite intriguing. For instance, in computations, if a different precision or a different computer is used, the computed values of statistical average can be different. In a laboratory experiment, measurements performed under nonidentical circumstances may yield different results.

- Y.-C. Lai, Z. Liu, G. Wei, and C.-H. Lai, "Shadowability of statistical averages in chaotic systems," *Physical Review Letters* **89**, 184101 (2002).

3.3 Objective 3: Signal enhancement using stochastic resonance for antijamming

3.3.1 Antijamming by stochastic resonance

An often encountered problem in many defense applications, such as communication and signal processing, is how to combat the influence of externally imposed, undesirable noise (jamming). A traditional and natural approach is to devise schemes to block or to significantly reduce the noise. Often, such a scheme involves sophisticated electronics, particularly in situations where the spectrum of the signal is contained entirely within that of the noise (in-band noise). An alternative approach to this well defined counter-jamming problem is by using the principle of stochastic resonance in nonlinear dynamical systems, as suggested by Dr. Arje Nachman at AFOSR. Broadly speaking, stochastic resonance means that certain performances of the system, such as the ability to detect periodic signals, can be enhanced by the presence of noise and be made optimal at certain nonzero noise levels. This phenomenon is rather counter-intuitive, but the key mechanism lies in the complex interplay between nonlinearity and noise.

We proposed a framework for antijamming by using stochastic resonance. The general philosophy is to make use of noise to counter noise. In particular, we constructed a signal processing unit by utilizing an array of excitable dynamical systems that exhibit stochastic resonance. Such a device can be simple and be built at low cost. The input signal consists of a desirable signal and jamming. A modulating noise signal is deliberately fed into the system. The system is so designed that at an optimal level of modulating noise, the complex interaction between nonlinearity and stochasticity yields an output signal with a much higher signal-to-noise ratio (SNR) than that of the input signal.

We investigated the performance of the stochastic-resonance-based scheme of antijamming with respect to periodic and aperiodic signals, and broad-band and narrow-band jamming as well.

- Z. Liu, Y.-C. Lai, and A. Nachman, "Enhancement of noisy signals by stochastic resonance," *Physics Letters A* **297**, 75-80 (2002).
- Z. Liu, Y.-C. Lai, and A. Nachman, "Enhancement of detectability of noisy signals by stochastic resonance in arrays," *International Journal of Bifurcation and Chaos*, accepted.
- Y.-C. Lai, Z. Liu, and A. Nachman, "Aperiodic stochastic resonance and phase synchronization," submitted to *Physics Letters A*.
- Y.-C. Lai, Z. Liu, A. Nachman, and L. Zhu, "Suppression of jamming in excitable systems by aperiodic stochastic resonance," submitted to *International Journal of Bifurcation and Chaos.*

3.3.2 Coherence resonance in coupled chaotic systems

Coherence resonance is a recently discovered phenomenon in which the degree of temporal regularity of the output of a nonlinear system in a noisy environment increases as the noise amplitude is increased and reaches maximum at an optimal noise level. Coherence resonance is distinct from the more common phenomenon of stochastic resonace in that the former concerns the timing while the latter deals with the amplitude of the output signal from a nonlinear system. In practical terms, if the performance of a detection scheme relies on the timing of a seemingly random output, then in the presence of jamming, applying certain intrinsic noise may in fact increase the regularity of the timing and consequently, enhance the performance.

We investigated the phenomenon of coherence resonace in the context of coupled nonlinear oscillators, which are commonly utilized in many electronic devices. We worked out a physical theory based on analyzing the Fokker-Planck equation and provided numerical evidence for the occurrence and quantification of coherence resonance in such systems.

- Z. Liu and Y.-C. Lai, "Coherence resonance in coupled chaotic oscillators," *Physical Review Letters* **86**, 4737-4740 (2001).
- Y.-C. Lai and Z. Liu, "Noise-enhanced temporal regularity in coupled chaotic oscillators," *Physical Review E* **64**, 066202(1-9) (2001).

3.3.3 Coherence resonance in non-bursting chaotic systems

Most existing works on coherence resonance had addressed excitable dynamical systems that typically generate bursting time series. While many nonlinear dynamical systems, nonchaotic or chaotic, can indeed exhibit bursting behaviors, many others do not. A question had been whether coherence resonance can occur in non-bursting dynamical systems. Our interest was in chaotic systems. Suppose there is a chaotic system that generates irregular but non-bursting signals, and suppose in a specific application, the temporal regularity of the signal is of interest. Would external noise help improve the temporal regularity of this signal? We argued theoretically that for a typical non-bursting chaotic system with many possible intrinsic time scales, noise can introduce a new time scale, or the external time scale. When the noise amplitude reaches a proper value, a resonant state can be reached in the sense that the external time scale matches one of the dominant internal time scales, leading possibly to coherence resonance. We obtained experimental evidence with a chaotic electronic circuit, the Chua's circuit. The implication is that noise can be beneficial, not only for bursting chaotic systems but also for non-bursting ones, so coherence resonance is expected to be ubiquitous in chaotic systems.

- L. Zhu, Y.-C. Lai, Z. Liu, and A. Raghu, "Can noise make non-bursting chaotic systems more regular?" *Physical Review E (Rapid Communications)* **66**, 015204(1-4) (2002).

3.3.4 Noise-induced enhancement of chemical reactions in chaotic flows

The interplay between noise and nonlinear dynamics had long been a topic of tremendous interest in statistical physics. While noise can be detrimental in many situations, it can also be beneficial through, for example, the mechanisms of stochastic and coherence resonances. Recently, a new area of interdisciplinary science emerged: active processes in nonlinear flows. Such processes can be chemical or biological, and are believed to be relevant to a large number of important problems in a variety of areas. Our work focused on the role of noise in active nonlinear processes. In particular, motivated by the problem of ozone production in atmospheres of urban areas, we investigated how noise influences a general type of chemical reaction, supported on a chaotic flow. To be as realistic as possible, we took into consideration important physical effects such as particle inertia and finite size. Our finding was that noise can enhance the rate of chemical reaction, in a manner similar to that of stochastic resonance. We provided numerical results and also a physical theory, suggesting that at a fundamental level, the resonant behavior is due to the interaction between noise and nonlinearity of the particle (Lagrangian) dynamics.

- Z. Liu, Y.-C. Lai, and J. M. Lopez, "Noise-induced enhancement of chemical reactions in chaotic flows," *Chaos* **12**, 417-425 (2002).
  This work was featured by the American Institute of Physics in AIP News. It was also named an "outstanding paper" in the journal CHAOS in 2002 by the AIP Annual Report.

3.4 Objective 4: Dynamics of semiconductor lasers and communicating with chaos

3.4.1 Chaotic transitions and low-frequency fluctuations in external-cavity semiconductor lasers

Semiconductor lasers offer many advantages not only due to their compact sizes but also because of their tremendous applications in various fields, particularly in optical data recording and optical-fiber communication. It has been known that the performances of these lasers, such as enhancement of the single longitudinal mode operation, spectral line narrowing, improved frequency stability, wavelength tunability etc., can be enhanced dramatically by a small amount of optical feedback. In applications where these properties are desirable, therefore, introducing optical feedback artificially by using additional optical elements can be quite advantageous. On the other hand, optical feedback is inevitable in virtually all realistic applications, which can be due to, for instance, reflections from fiber facets when radiation is coupled into a fiber. Semiconductor lasers subject to optical feedback are called external-cavity lasers. From the standpoint of dynamics, optical feedbacks introduce a time delay to the reinjected field which in turn, creates an infinite number of degrees of freedom, making the phase-space dimension of the underlying dynamical system infinite. This makes the analysis and understanding of external-cavity semiconductor lasers an extremely challenging problem.

While optical feedback at low levels can indeed be advantageous, the performances of semiconductor lasers can be degraded significantly when the feedback is at moderate or high levels. In particular, at high feedback levels, a semiconductor laser can enter the so-called coherence collapse regime where the optical linewidth increases drastically. At moderate feedback levels, when the pumping current is close to the solitary threshold, the laser intensity can exhibit sudden dropouts at irregular times, followed by a slow and gradual recovery after each dropout. The average frequency of the dropouts is typically at the MHz-level, which is several orders of magnitude smaller than the frequency of the solitary laser relaxation oscillation. This dropout phenomenon is thus called low-frequency fluctuations (LFFs), which poses a serious difficulty in many applications where a sustained laser power is desirable.

Semiconductor lasers are of tremendous value to a large variety of applications that are important for the Air Force missions. Suggested by Drs. A. Gavrielides and V. Kovanis from the Air Force Research Laboratory at the Kirtland Air Force Base, we studied chaotic transitions and low-frequency fluctuations in external-cavity semiconductor lasers by numerical integration of the Lang-Kobayashi equations. Our findings were: (1) At moderate feedback levels, the system evolves around the remains of attractors that were first created in place of a cascade of external cavity modes and then merged together in a sequence of chaotic transitions; (2) Depending on the feedback strength, the highest gain mode is either unattainable by the system or can be reached after a chaotic transient. The implication is that if the highest gain mode is unreachable, then the output of the laser exhibits LFFs. If, however, the highest gain mode is reachable, then LLFs are only a transient as the laser can operate in a stable, high gain mode despite the presence of delayed optical feedback. We provided explanations for these observations from the prespective of chaos theory.

- R. L. Davidchack, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Dynamical origin of low-frequency fluctuations in external cavity semiconductor lasers," *Physics Letters A* **267**, 350-356 (2000).

- R. L. Davidchack, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Chaotic transitions and low-frequency fluctuations in semiconductor lasers with optical feedback," *Physica D* **145**, 130-143 (2000).

3.4.2 Regular dynamics associated with low-frequency fluctuations in external-cavity semiconductor lasers

It is commonly believed that the dynamics responsible for low-frequency luctuations (LFFs) in external cavity semiconductor lasers is stochastic or chaotic. A common approach to address the origin of LFFs had been to investigate the dynamical behavior of, and the interaction among, various external cavity modes (ECMs) in the Lang-Kobayashi (LK) paradigm. The ECM framework had been, however, inadequate in explaining many features of the LFFs. We proposed a framework for understanding LFFs based on a different set of fundamental solutions of the LK equations. In particular, we presented strong numerical evidence and a heuristic argument indicating that the underlying "backbone" dynamics of LFFs can be regular (quasiperiodic or periodic), which is characterized by a sequence of time-locked pulses and can actually be observed when: (1) the feedback level is moderate, (2) pumping current is below solitary threshold, and (3) the linewidth enhancement factor is relatively large. These results had implications in the understanding and applications of external cavity semiconductor lasers. For instance, one might be interested in applying control to eliminate LFFs. Knowing that the underlying dynamics has a regular backbone despite irregularity of LFFs, one can attempt strategies that are different from the commonly utilized approach of controlling chaos.

- R. L. Davidchack, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Regular dynamics of low-frequency fluctuations in semiconductor lasers with optical feedback," *Physical Review E* **63**, 056206(1-6) (2001).

3.4.3 A dynamical-system approach to low-frequency fluctuations in external-cavity semiconductor lasers

We investigated the dynamical origin of LFFs by utilizing a simplified, three-dimensional model derived from the infinite- dimensional Lang-Kobayashi (LK) equations. The simplified model preserves the dynamical properties of the external-cavity modes (ECMs) and antimodes which play a fundamental role in the generation of LFFs. This model yields a clear picture of the dynamical origin of the LFFs. In particular, we showed that, in the absence of noise, LFFs are a consequence of the dynamical interactions among different ECMs and antimodes. When a small amount of noise is present, LFFs result from an intermittent switching of trajectories among different coexisting attractors in the phase space. The presence of double peaks in the distribution of power dropout times, which had been observed in experiments, was explained, and a scaling relation was obtained between the average switching time and the noise strength.

- A. Prasad, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Low-frequency fluctuations in external-cavity semiconductor lasers: understanding based on a reduced Lang-Kobayashi model," *Journal of Optics B: Quantum and Semiclassical Optics* **3**, 242-250 (2001).

3.4.4 Controlling low-frequency fluctuations in external-cavity semiconductor lasers

We proposed a time-delay coupling scheme to modulate the amplitude of external-cavity semiconductor lasers so as to control the LFFs. In particular, we demonstrated, by making use of the principle of amplitude death, that in a suitable coupling scheme, semiconductor lasers can be harnessed in such a desirable way that power dropouts can be eliminated completely. The scheme provided a possible solution to controlling LFFs in external-cavity semiconductor lasers, which may pave the way for more desirable and broader applications of these lasers.

We also discovered the phenomenon of multiple coexisting attractors in coupled external-cavity semiconductor lasers and studied the complicated structure of basins of attraction of these attractors.

- A. Prasad, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Complicated basins in coupled external-cavity semiconductor lasers," *Physics Letters A* **314**, 44-50 (2003).

- A. Prasad, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Amplitude modulation in coupled external-cavity semiconductor lasers," *Physics Letters A*, in press.

3.4.5 Communicating with chaos using two-dimensional symbolic dynamics

Nonlinear digital communication (communicating with chaos) had become a topic of interest to researchers in defense laboratories. We explored various theoretical and computational issues in nonlinear digital communication. Existing works illustrating the principle of communicating with chaos all had utilized chaotic systems exhibiting one-dimensional dynamics. We investigated the possibility of communicating with chaos by using more realistic dynamical systems described by three-dimensional flows. The major difficulty was how to specify a generating partition so that a good symbolic dynamics can be defined. We proposed a solution to use hyperbolic chaotic saddles embedded in the chaotic attractor for message encoding. Potentially, our results implied that a large variety of practically usable chaotic systems can be utilized as effective information sources.

- Y.-C. Lai, E. Bollt, and C. Grebogi, "Communicating with chaos using two-dimensional symbolic dynamics," *Physics Letters A* **255**, 75-81 (1999).

3.4.6 Communicating with transient chaos

We explored communicating with transient chaos naturally arising in nonlinear systems. Dynamically, transient chaos is caused by nonattracting chaotic saddles. We argued that there are two major advantages when trajectories on chaotic saddles are exploited for communication: (1) the channel capacity can in general be large; and (2) the influence of channel noise can be reduced. We devised a control scheme for realizing communication and studied practical examples of encoding digital messages.

- Y.-C. Lai "Encoding digital information using transient chaos," *International Journal of Bifurcation and Chaos* **10**, 787-795 (2000).

3.4.7 Estimating generating partitions of chaotic systems by unstable periodic orbits

An outstanding problem in chaotic dynamics was to specify generating partitions for symbolic dynamics in dimensions larger than one. Being of fundamental importance to the study of chaotic dynamics, finding generating partitions is also critical for important technological applications such as communicating with chaos. It had been known that the infinite number of unstable periodic orbits embedded in the chaotic invariant set provides sufficient information for estimating the generation partition. We proposed a general, dimension-independent, and efficient approach for this task based on optimizing a set of proximity functions defined with respect to periodic orbits. Our algorithm allowed us to obtain, for the first time, generating partitions for chaotic systems such as the Ikeda-Hammel-Jones-Moloney map.

- R. L. Davidchack, Y.-C. Lai, E. Bollt, and M. Dhamala, "Estimating the generating partition of chaotic systems by unstable periodic orbits," *Physical Review E* **61**, 1353-1356 (2000).

3.4.8 Validity of threshold-crossing analysis of symbolic dynamics from chaotic time series

Symbolic dynamics is a convenient tool to describe complicated time evolution of chaotic dynamical systems, and it provides a natural link between chaotic dynamics and information theory, on which the idea of utilizing chaotic systems to encode digital information, or communicating with chaos, was based. A good symbolic dynamical representation requires a one-to-one correspondence to the phase-space dynamics; the partition that defines distinct symbols has to be generating. On the experimental side, there was an increasing interest in chaotic symbolic dynamics as well. A common practice had been to apply the threshold-crossing method, i.e., to define a rather arbitrary partition, so that distinct symbols can be defined from measured time series. There are two reasons for the popularity of the threshold-crossing method: (1) it is extremely difficult to locate the generating partition from chaotic data, and (2) threshold-crossing is a physically intuitive and natural idea. It is thus of paramount interest, from both the theoretical and experimental points of view, to understand how misplaced partitions affect the goodness of the symbolic dynamics such as the amount of information that can be extracted from the data.

We investigated the consequence of misplaced partitions in chaotic systems. Specifically, we addressed how the topological entropy, perhaps one of the most important dynamical invariants that one intends to compute from symbolic dynamics, behaves as a parameter characterizing the amount of misplaced partition is changed. We found the topological entropy as a function of the parameter to be devil's staircase-like, but surprisingly nonmonotone. We established our results by numerical computations for one- and two-dimensional maps, and by a rigorous analysis for the tent map which is a good topological model for one-dimensional one-hump maps. The main implication of our results was that the threshold-crossing technique typically yields misleading conclusions about the dynamics generating the data, and therefore one should be extremely cautious when attempting to understand the underlying system from a misrepresented symbolic dynamics.

- E. Bollt, T. Stanford, Y.-C. Lai, and K. Zyczkowski, "Validity of threshold-crossing analysis of symbolic dynamics from chaotic time series," *Physical Review Letters* **85**, 3524-3527 (2000).

- E. Bollt, T. Stanford, Y.-C. Lai, and K. Zyczkowski, "What symbolic dynamics do we get with a misplaced partition? - On the validity of threshold crossings analysis of chaotic time series," *Physica D* **154**, 259-286 (2001).

3.5 Achievements on related research

3.5.1 Modeling of chaotic systems

Scientists and engineers rely heavily on mathematical models to understand natural phenomena. Usually, for a particular process, data from laboratory experiments or from observations are analyzed and, together with physical laws, a mathematical model of the process is formulated. In fact, this is done for a large variety of processes in fields such as physics, chemistry, biology, ecology, and engineering. The models are then used to understand the particular process, to make predictions, and to control its dynamics. There are two important classes of models. The first class is the deterministic dynamical systems and they evolve the relevant physical variables in time according to a set of prescribed rules. The second one is stochastic models, models which involve some kind of random process and, consequently, for these models, only statistical averages regarding properties of the system can be obtained from the model. The conventional wisdom about statistical models is that they deal with situations where random noise is influential or systems that involve a large number of degrees of freedom such as those arising in statistical physics.

We discovered a class of deterministic models for chaotic systems which, in spite of being deterministic, yield only statistically relevant information about their dynamical variables. In particular, systems of coupled chaotic oscillators occur in many situations of physical and biological interest. They can also come from discretization of nonlinear partial differential equations. We argued that severe modeling difficulties are possible in the sense that no modeling is able to produce reasonably long solutions that are realized by nature or by the original nonlinear differential equations. We obtained theoretical and numerical evidence that this obstruction to modeling may occur when a coupling, however small, is present.

The implication of our result is as follows. Say one constructs a natural system of coupled chaotic oscillators in a laboratory, and one measures a trajectory. Then no trajectory of reasonable length from the mathematical model of the natural system is close to the measured trajectory. The difficulty to model this natural process is a consequence of the inexactitude of the model given by the inevitable random disturbances and imperfections of the model such as various approximations used in the model-building process. If the model is an approximation to the natural process, as indeed it is, due to imperfections of the natural system, no model can produce trajectories of reasonable length that are close to trajectories of the actual system of coupled oscillators. Thus, one should exercise some care when studying and interpreting results from models of coupled chaotic oscillators. Often, the only long-term meaningful results one can trust are the statistical invariants obtained by simulating a large number of trajectories of the model. In laboratory experiments involving coupled chaotic oscillators, it might only make sense to work *directly* with measured time series instead of a mathematical model when attempting to understand the long term behavior of the system, even if the model is built upon physical laws and is considered to be reasonable.

- Y.-C. Lai and C. Grebogi, "Modeling of coupled chaotic oscillators," *Physical Review Letters* **82**, 4803-4806 (1999).

- Y.-C. Lai, C. Grebogi, and J. Kurths, "Modeling of deterministic chaotic systems," *Physical Review E* **59**, 2907-2910 (1999).

- Y.-C. Lai, D. Lerner, K. Williams, and C. Grebogi, "Unstable dimension variability in coupled chaotic oscillators," *Physical Review E* **60**, 5445-5454 (1999).

- Y.-C. Lai and C. Grebogi, "Obstruction to deterministic modeling of chaotic systems with invariant manifold," *International Journal of Bifurcation and Chaos* **10**, 683-693 (2000).

- Y. Do, Y.-C. Lai, Z. Liu, and E. J. Kostelich, "Universal and nonuniversal scaling in shadowing dynamics of nonhyperbolic chaotic systems with unstable dimension variability," *Physical Review E (Rapid Communications)* **67**, 035202 (1-4) (2003).

3.5.2 Towards complete detection of unstable periodic orbits in chaotic systems

An outstanding problem in chaotic dynamics had been how to compute complete sets of unstable periodic orbits (UPOs), where are perhaps the most fundamental building blocks of invariant sets in chaotic dynamical systems because, many measureable quantities of physical interest can be related to the dynamical properties of the set of infinite number of UPOs embedded in the chaotic set. We made a progress in this direction. In particular, we proposed an algorithm for detecting UPOs in general chaotic systems, which is many orders of magnitude more efficient than ALL existing methods. We were able to argue, rigorously for one-dimensional chaotic maps, that our algorithm is indeed capable of yielding ALL UPOs up to periods that are limited by the computer round-off. In high dimensions, we had convincing numerical evidence for complete detection of UPOs by our algorithm.

- R. L. Davidchack and Y.-C. Lai, "An efficient algorithm for detecting unstable periodic orbits in chaotic systems," *Physical Review E* **60**, 6172-6175 (1999).

- R. L. Davidchack, Y.-C. Lai, A. Klebanoff, and E. M. Bollt, "Toward complete detection of unstable periodic orbits in chaotic systems," *Physics Letters A* **287**, 99-104 (2001).

3.5.3 Riddling in chaotic systems

Most existing works on riddling had assumed that the underlying dynamical system possesses an invariant subspace that usually results from a symmetry. In realistic applications of chaotic systems, however, there exists no perfect symmetry. We investigated the consequences of symmetry-breaking on riddling. In particular, we considered smooth *deterministic* perturbations that destroy the existence of invariant subspace and identify, as a symmetry-breaking parameter is increased from zero, two distinct bifurcations. In the first case, the chaotic attractor in the invariant subspace is transversely stable so that it basin is riddled. We found that a bifurcation from riddled to fractal basins can occur in the sense that an arbitrarily small amount of symmetry-breaking can replace the riddled basin by fractal ones. We called it a *catastrophe of riddling.* In the second case where the chaotic attractor in the invariant subspace is transversely unstable so that there is no riddling in the unperturbed system, the presence of a symmetry-breaking, no matter how small, can immediately create fractal basins in the vicinity of the original invariant subspace. This is a smooth-fractal basin boundary metamorphosis. We analyzed the dynamical mechanisms for both catastrophes of riddling and basin boundary metamorphoses, derived scaling laws to characterize the fractal basins induced by symmetry-breaking, and provided numerical confirmations. The main implication of our results is that, while riddling is robust against perturbations that preserve the system symmetry, riddled basins of chaotic attractors in the invariant subspace, on which most existing works were focused, are *structurally unstable* against symmetry-breaking perturbations. A striking consequence is that riddling is not physically observable, in contrast to previous claims. What can be observed in laboratory experiments is fractal basins that appear like riddled ones.

- Y.-C. Lai and C. Grebogi, "Riddling of chaotic sets in periodic windows," *Physical Review Letters* **83**, 2926-2929 (1999).

- Y.-C. Lai, "Catastrophe of riddling," *Physical Review E (Rapid Communications)* **62**, R4505-R4508 (2000).

- Y.-C. Lai, "Pseudo-riddling in chaotic systems," *Physica D* **150**, 1-13 (2001).

- Y.-C. Lai and V. Andrade, "Catastrophic bifurcation from riddled to fractal basins," *Physical Review E* **64**, 056228(1-16) (2001).

3.5.4 Other related research on nonlinear dynamics

We also published on the following topics:

- Transition to high-dimensional chaos;
- Analytic signal representation of chaotic time series;
- Complexity and unstable periodic orbits in chaotic systems;
- Controlling transient chaos in deterministic flows;
- Natural measure and unstable periodic orbits of transient chaos;
- Characterization of bifurcation to high-dimensional chaos by unstable periodic orbits;
- Detection of unstable periodic orbits from experimental transient chaotic time series;
- Scaling analysis of crisis in chaotic systems;
- Synchronization in complex networks.

4 Personnel Supported and Theses Supervised by PI

4.1 Personnel Supported

The following people received salary from the PECASE in various time periods.

- **Faculty (partial summer salary)**: Ying-Cheng Lai (PI), Professor of Mathematics, Professor of Electrical Engineering, Affiliated Professor of Physics
- **Post-Doctoral Fellows (full- or part-time appointments)**

1. Ruslan Davidchack
2. Zong-Hua Liu
3. Adilson E. Motter
4. Alessandro de Moura
5. Takashi Nishikawa
6. Awadhesh Prasad

- **Graduate Students (part-time appointments)**

1. Victor Andrade, University of Kansas
2. Mukeshwar Dhamala, University of Kansas
3. Younghae Do, Arizona State University

4. Mary Ann Harrison, University of Kansas
5. Arvind Raghu, Arizona State University
6. Lonnie Sauter, University of Kansas
7. Saeed Taherion, University of Kansas
8. Bin Xu, Arizona State University
9. Liqiang Zhu, Arizona State University

- **Other**: Thomas Erneux, Air Force Scientific Consultant.

4.2 Theses Supervised by PI

- **Ph.D. Theses**

1. Tolga Yalcinkaya, Physics, University of Kansas, 1998. Dissertation: *Phase characterization and controlling chaos in deterministic flows.* Immediate job after Ph.D. - Scientist, Advanced Research Division, Sprint.
2. Saeed Taherion, Physics, University of Kansas, 1999. Dissertation: *Experimental and numerical studies of synchronization in nonlinear chaotic oscillators and effect of filtering on topological entropy.* Immediate job after Ph.D. - Post-Doctoral Fellow, Department of Electrical and Computer Engineering, University of Kansas.
3. Mukeshwar Dhamala, Physics, University of Kansas, 2000. Dissertation: *Transient chaos.* Immediate job after Ph.D. - Post-Doctoral Fellow, Georgia Institute of Technology.
4. Lonnie Sauter, Mathematics, University of Kansas, 2001. Dissertation: *Generalized synchronism, low-dimensional chaos, and phase synchronization in coupled chaotic systems.* Immediate job after Ph.D. - Senior Network Engineer, Sprint.
5. Mary Ann Harrison, Physics, University of Kansas, 2001. Dissertation (with honor): *On-off intermittency and patterning in spatially extended dynamical systems.* Immediate job after Ph.D - Research Scientist, Flint Hill Scientific, Kansas.
6. Victor Andrade, Physics, University of Kansas, 2002. Dissertation: *Superpersistent chaotic transient and bifurcation from riddled to fractal basins.* Immediate job after Ph.D - Post-Doctoral Fellow, Department of Physics and Astronomy, University of Kansas.

- **Master Theses**

1. Mukeshwar Dhamala, Physics, University of Kansas, 1999. Thesis: *Controlling transient chaos in deterministic flows with applications to electrical power systems and ecology.*
2. Mary Ann Harrison, Physics, University of Kansas, 1999. Thesis: *Transition to high-dimensional chaos.*
3. Arvind Raghu, Electrical Engineering, Arizona State University, 2001. Thesis: *Double coherence resonance in chaotic systems and experimental observation.*
4. Bin Xu, Electrical Engineering, Arizona State University, 2003. Thesis: *Shadowing of statistical averages in chaotic systems.*

5 List of Publications

5.1 Refereed Journals

Journal/# of Papers

Journal	**# of papers**
Physical Review Letters	10
Physical Review E (Rapid Communications)	14
Physical Review E (Regular Articles)	20
Physical Review E (Brief Reports)	3
Physics Letters A	8
Physica D	4
Chaos	2
International Journal of Bifurcation and Chaos	8
Physics Reports	1
Discrete Dynamical Systems	1
Journal of Optics B	1
Chaos, Solitons, and Fractals	1
Submitted	7
Total	**80**

1. Y.-C. Lai, "Analytic signals and transition to chaos in deterministic flows," *Physical Review E (Rapid Communications)* **58**, R6911-R6914 (1998).

2. M. Dhamala and Y.-C. Lai, "Controlling transient chaos in deterministic flows with applications to electrical power systems and ecology," *Physical Review E* **59**, 1646-1655 (1999).

3. Y.-C. Lai, C. Grebogi, and J. Kurths, "Modeling of deterministic chaotic systems," *Physical Review E* **59**, 2907-2910 (1999).

4. M. Harrison and Y.-C. Lai, "A route to high-dimensional chaos," *Physical Review E (Rapid Communications)* **59**, R3799-R3802 (1999).

5. Y.-C. Lai, "Unstable dimension variability and complexity in chaotic systems," *Physical Review E (Rapid Communications)* **59**, R3807-R3810 (1999).

6. Y.-C. Lai, K. Zyczkowski, and C. Grebogi, "Universal behavior in the parameter evolution of chaotic saddles," *Physical Review E* **59**, 5261-5265 (1999).

7. Y.-C. Lai, E. Bollt, and C. Grebogi, "Communicating with chaos using two-dimensional symbolic dynamics," *Physics Letters A* **255**, 75-81 (1999).

8. Y.-C. Lai and C. Grebogi, "Modeling of coupled chaotic oscillators," *Physical Review Letters* **82**, 4803-4806 (1999).

9. S. Taherion and Y.-C. Lai, "Observability of lag synchronization in coupled chaotic oscillators," *Physical Review E (Rapid Communications)* **59**, R6247-R6250 (1999).

10. Y.-C. Lai, "Transient fractal behavior in snapshot attractors of driven chaotic systems," *Physical Review E* **60**, 1558-1562 (1999).

11. Y.-C. Lai and C. Grebogi, "Riddling of chaotic sets in periodic windows," *Physical Review Letters* **83**, 2926-2929 (1999).

12. T. Kapitaniak, Y.-C. Lai, and C. Grebogi, "Metamorphosis of chaotic saddles," *Physics Letters A* **259**, 445-450 (1999).

13. Y.-C. Lai, D. Lerner, K. Williams, and C. Grebogi, "Unstable dimension variability in coupled chaotic oscillators," *Physical Review E* **60**, 5445-5454 (1999).

14. R. L. Davidchack and Y.-C. Lai, "An efficient algorithm for detecting unstable periodic orbits in chaotic systems," *Physical Review E* **60**, 6172-6175 (1999).

15. M. Dhamala and Y.-C. Lai, "Unstable periodic orbits and the natural measure of nonhyperbolic chaotic saddles," *Physical Review E* **60**, 6176-6179 (1999).

16. T. Kapitaniak, Y.-C. Lai, and C. Grebogi, "Blowout bifurcation of chaotic saddles," *Discrete Dynamics in Nature and Society* **3**, 9-13 (1999).

17. Y.-C. Lai, "Abrupt bifurcation to chaotic scattering with discontinuous change in fractal dimension," *Physical Review E (Rapid Communications)* **60**, R6283-R6286 (1999).

18. R. L. Davidchack, Y.-C. Lai, E. Bollt, and M. Dhamala, "Estimating the generating partition of chaotic systems by unstable periodic orbits," *Physical Review E* **61**, 1353-1356 (2000).

19. V. Andrade, R. L. Davidchack, and Y.-C. Lai, "Noise scaling in phase synchronization of coupled chaotic oscillators," *Physical Review E* **61**, 3230-3233 (2000).

20. S. Boccaletti, C. Grebogi, Y.-C. Lai, H. Mancini, and D. Maza, "Control of chaos: theory and applications," *Physics Reports* **329**, 103-197 (2000).

21. R. L. Davidchack, Y.-C. Lai, A. Gavrielides, and V. Kovanis, "Dynamical origin of low-frequency fluctuations in external cavity semiconductor lasers," *Physics Letters A* **267**, 350-356 (2000).

22. K. G. Szabo, Y.-C. Lai, T. Tél, and C. Grebogi, "Topological scaling and gap filling at crisis," *Physical Review E* **61**, 5019-5032 (2000).

23. Y.-C. Lai and C. Grebogi, "Obstruction to deterministic modeling of chaotic systems with invariant manifold," *International Journal of Bifurcation and Chaos* **10**, 683-693 (2000).

24. M. Dhamala, Y.-C. Lai, and E. J. Kostelich, "Detecting unstable periodic orbits from transient chaotic time series," *Physical Review E* **61**, 6485-6489 (2000).

25. K. Zyczkowski and Y.-C. Lai, "Devil-staircase behavior of dynamical invariants in chaotic scattering," *Physica D* **142**, 197-216 (2000).

26. Y.-C. Lai "Encoding digital information using transient chaos," *International Journal of Bifurcation and Chaos* **10**, 787-795 (2000).

27. R. L. Davidchack and Y.-C. Lai, "Characterization of transition to chaos with multiple positive Lyapunov exponents by unstable periodic orbits," *Physics Letters A* **270**, 308-313 (2000).

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75. Y.-C. Lai, Z. Liu, A. Nachman, and L. Zhu, "Suppression of jamming in excitable systems by aperiodic stochastic resonance," submitted to *International Journal of Bifurcation and Chaos.*

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79. Y. Do and Y.-C. Lai, "Statistics of shadowing time in nonhyperbolic chaotic systems with unstable dimension variability," submitted to *Physical Review E.*

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5.2 Books/Book Chapters

1. C. Grebogi, Y.-C. Lai, and S. Hayes, "Control and applications of chaos," in *Visions of Nonlinear Science in the 21st Century*, edited by J. L. Huertas, W.-K. Chen, and R, N. Madan (World Scientific, 1999).

2. Y.-C. Lai, "Chaotic dynamics: introduction and recent developments," pp. 295-306 in *Recent Advances & Cross-Century Outlooks in Physics*, edited by P. Chen and C.-Y. Wong (World Scientific, 2000).

3. Y.-C. Lai, V. Andrade, R. L. Davidchack, and S. Taherion, "Experimental manifestations of phase and lag synchronizations in coupled chaotic systems," in *Proceeding of the 5th Experimental Chaos Conference*, edited by L. Pecora *et al.* (World Scientific, 2000).

4. Y.-C. Lai and C. Grebogi, "Necessity of statistical modeling of deterministic chaotic systems," pp. 531-542 in *Statistical Physics (Third Tohwa University International Conference)*, edited by M. Tokuyama and H. E. Stanley (American Institute of Physics, 2000).

6 Interactions/Transitions

The PI provided scientific consultation to Dr. A. Gavrielides' and Dr. M. Harrison's groups at AFRL at Kirtland Air Force Base, and to Dr. D. H. Hughes at AFRL in Rome, NY.

During the project period, the PI presented 50 invited lectures, seminars, and colloquia at various conferences and universities, as follows.

- Invited Talks at Conferences and Symposia (one-hour long unless otherwise specified)

1. "Modeling of coupled chaotic oscillators," International Workshop on Lattice Dynamics, National Chiao Tong University, Taiwan, June 27, 1998.

2. "Scaling of encounter probability with unstable periodic orbits in time series," International Workshop on Experimental Detection of Periodic Orbits, George Mason University, July 11, 1998.

3. "Modeling of coupled chaotic oscillators," Nonlinear Optics Workshop, University of Arizona, September 24, 1998. (*30 minutes*)

4. Short course on chaotic dynamics: (1) Controlling chaos; (2) Communicating with chaos; (3) Chaotic scattering; (4) Hamiltonian chaos and quantum chaos; (5) Crises; (6) Quasiperiodic systems; (7) Riddling; (8) Effect of noise on chaotic dynamics; (9) Phase dynamics of chaos; (10) Modeling of chaotic systems; (11) Unstable periodic orbit theory; and (12) Chaotic time series analysis, National Chung Cheng University, Taiwan, November 27 - December 8, 1998 (*12 hours*)

5. "Transition to strange nonchaotic and chaotic attractors," International Workshop on *Beyond Quasiperiodicity, Complex Dynamics and Structure*, Dresden, Germany, January 11, 1999.

6. "Communicating with chaos," MURI Workshop on *Communicating with Chaos*, University of California, San Diego, January 25, 1999. (*90 minutes*)

7. "Chaotic scattering," First Overseas Chinese Physics Association Conference on *Recent Advances and Cross-Century Outlooks in Physics: Interplay between Theory and Experiment*, Atlanta, March 19, 1999.

8. "Scaling law for detecting unstable periodic orbits from transient chaos," Symposium on *Frontiers in Nonlinear Dynamics and Neurodynamics*, Georgia Institute of Technology, March 20, 1999.

9. Fifth SIAM Conference on Dynamical Systems, Snowbird, May 21-15, 1999, two invited minisymposium talks: (1) "Modeling of coupled chaotic systems" and (2) "Communicating with chaos using two-dimensional symbolic dynamics."

10. "Experimental study of phase and lag synchronizations in coupled chaotic systems," the Fifth Experimental Chaos Conference, Orlando, Florida, June 29, 1999. (*30 minutes*)

11. "Estimating generating symbolic partitions by unstable periodic orbits," Nonlinear Optics Workshop, University of Arizona, September 17, 1999. (*30 minutes*)

12. "Riddling and on-off intermittency," International School on *Space Time Chaos: Characterization, Control, and Synchronization*, Pamplona, Spain, June 23, 2000. (*Two hours*)

13. SIAM Pacific Rim Dynamical Systems Conference, Maui, Hawaii, August 10-15, 2000, three invited minisymposium talks: (1) "Computing the correlation dimension from time series in weakly coupled spatiotemporal chaotic systems," (2) "Intermittency in chaotic rotations," and (3) "Effect of noise on phase synchronizatin and experimental observation of lag synchronization in coupled chaotic systems."

14. "Chaotic scattering and the dynamics of optical microlasing cavities," AFOSR Nonlinear Optics Workshop, University of Arizona, Tucson, September 22, 2000. (*30 minutes*)

15. "Estimating generating symbolic partition in chaotic systems by unstable periodic orbits," Southwest Dynamical Systems Conference, University of Southern California, November 19, 2000. (*30 minutes*)

16. "Superpersistent chaotic transients," International Workshop on *Physics of Information and Synchronization in Stochastic Dynamics*, Max-Planck Institute for Physics of Complex Systems, Dresden, Germany, April 2, 2001. (*30 minutes*)

17. Sixth SIAM Conference on Dynamical Systems, Snowbird, Utah, May 19-24, 2001, three invited minisymposium talks: (1) "Catastrophe of riddling and implications to shadowing," (2) "Superpersistent chaotic transients: theory and experiments," and (3) "Low-dimensional chaos in high-dimensional dynamical systems: how does it occur?"

18. "Noise-induced enhancement of chemical reactions in chaotic flows," International Workshop on Active Chaotic Flows, Los Alamos National Laboratory, May 29, 2001.

19. "Effect of filtering on information-carrying capacity of chaotic signals," International Workshop on *Control, Communication, and Synchronization of Chaotic Systems*, Max-Planck Institute for Physics of Complex Systems, Dresden, Germany, October 16, 2001. (*30 minutes*)

20. "Growing and small-world networks," Second Asian/Pacific Dynamics Days Conference, Hangzhou, China, August 9, 2002.

21. "Stochastic resonance induced by coherence resonance: an example in active chaotic flows," International Workshop on *Chemical and Biological Activities in Flows*, Max-Planck Institute for Physics of Complex Systems, Dresden, Germany, September 25, 2002.

22. "Noise-induced chaos in electronic circuits," AFOSR/MURI Chaos Meeting, University of Maryland at College Park, April 25, 2003.

23. "Structure and dynamics of complex networks," *Networks - Structure, Dynamics, and Function*, Los Alamos National Laboratory Center for Nonlinear Studies 23rd Annual Conference, Santa Fe, May 12, 2003. (*30 minutes*)

24. SIAM Conference on Dynamical Systems, Snowbird, Utah, May 26-31, 2003, two invited minisymposium talks: (1) "Coherence resonance in chaotic systems," and (2) "Shadowing of statistical averages in chaotic systems."

25. "Complex Networks," AFOSR Software & Systems and Fusion Annual Workshop, Syracuse, June 5, 2003. (*15 minutes*)

- Invited Colloquia and Seminars at Universities

26. "Communicating with chaos," Colloquium, Department of Physics, University of Missouri at Kansas City, October 2, 1998.

27. "Transient chaos," Colloquium, Department of Mathematics, Arizona State University, April 29, 1999.

28. "Modeling and complexity of coupled chaotic systems," Colloquium, Department of Electrical Engineering, Arizona State University, April 30, 1999.

29. "Abrupt bifurcation to chaotic scattering," Nonlinear Dynamics Seminar, Department of Mathematics, Arizona State University, January 28, 2000.

30. "Chaotic scattering," Condensed Matter Physics Seminar, Department of Physics, Arizona State University, February 18, 2000.

31. "Chaotic scattering," Colloquium, Department of Mathematics, U.S. Naval Academy, October 13, 2000.

32. "Unstable periodic orbits and generating partitions in chaotic systems," Colloquium, Krasnow Institute for Advanced Studies, George Mason University, December 8, 2000.

33. "Riddling in chaotic systems," Joint Colloquium, Departments of Physics and Computational Science, National University of Singapore, August 22, 2001.

34. "Riddling, superpersistent chaotic transients, and small worlds," Colloquium, Whiting School of Engineering, Johns Hopkins University, October 29, 2001.

35. "Noise and active processes in chaotic flows," Seminar, Department of Computational Science, National University of Singapore, November 28, 2001.

36. "Transition to chaos in continuous-time random dynamical systems," Seminar, Department of Computational Science, National University of Singapore, December 5, 2001.

37. "Growing and small-world networks," Seminar, Department of Physics, Beijing Normal University, China, July 29, 2002.

38. "Chaotic scattering in open Hamiltonian systems and applications," Seminar, Department of Mathematics, Jilin University, China, August 1, 2002.

39. "Growing and small-world networks," Seminar, Department of Physics, Jilin University, China, August 2, 2002.

40. "Transition to chaos in random dynamical systems," Seminar, Department of Mathematics, Nanjing University, China, August 5, 2002.

41. "Shadowability of statistical averages in chaotic systems," Seminar, Center for Nonlinear Dynamics, Georgia Institute of Technology, November 11, 2002.

42. "Small-world and growing networks," Seminar, National Center for Theoretical Sciences, Taiwan, November 21, 2002.

43. "Transition to chaos in random dynamical systems," Seminar, National Center for Theoretical Sciences, Taiwan, November 21, 2002.

44. "Introduction to growing networks," Particle Physics Seminar, Department of Physics, National Tsing Hua University, Taiwan, November 25, 2002.

45. "Superpersistent chaotic transients and experimental verification," Colloquium, Department of Applied Mathematics, National Chiao Tung University, Taiwan, November 26, 2002.

46. "Coherence resonance in chaotic systems," Condensed Matter Physics Seminar, Department of Physics, National Tsing Hua University, Taiwan, November 26, 2002.

47. "Coherence resonance in chaotic systems," Seminar, National Center for Theoretical Sciences, Taiwan, November 28, 2002.

48. "Chaotic scattering in open Hamiltonian systems and applications," Seminar, National Center for Theoretical Sciences, Taiwan, November 28, 2002.

49. "Can statistical averages of chaotic systems be computed or measured reliably?" Colloquium, Department of Physics, National University of Singapore, March 20, 2003.

50. "Superpersistent chaotic transients," Nonlinear Dynamics Seminar, Department of Physics and Tamesek Laboratory, National University of Singapore, March 21, 2003.

7 Honors

1. This award (1997 Air Force PECASE).

2. In November 1999, The PI was elected as a Fellow of the American Physical Society with the citation *For his many contributions to the fundamentals of nonlinear dynamics and chaos.*